



SOLVING EQUATIONS BY COMPLETING THE SQUARE

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There are several ways to solve an equation by completing the square. Two methods begin by dividing the equation by a number that will change the "lead coefficient" to 1. Another method completes the square within a set of parentheses. In all cases, the goal of completing the square is to rewrite a polynomial as a quantity raised to the power of "2", the square. Listed below are three methods: a). move the constant term, b). do not move the constant term, c). complete within parentheses.

Method: lead coefficient of 1 and move the constant term.

$$\text{Solve: } 3x^2 - 3x - 2x + 2 = -17x + 8$$

Step 1:

Use algebra to simplify both sides of the equation.

$$3x^2 - 3x - 2x + 2 = -17x + 8$$

$$3x^2 - 5x + 2 = -17x + 8$$

Step 2:

Again, use algebra to write the equation in standard form: a polynomial on the left side of the equation and a zero on the right. The polynomial should be listed in descending powers: the highest power first and the constant last.

$$3x^2 - 5x + 2 = -17x + 8$$

$$3x^2 - 5x + 2 - 8 = -17x + 8 - 8$$

$$3x^2 - 5x - 6 = -17x$$

$$3x^2 - 5x - 6 + 17x = -17x + 17x$$

$$3x^2 - 5x + 17x - 6 = 0$$

$$3x^2 + 12x - 6 = 0$$

Step 3:

If the lead coefficient is NOT the number 1, then divide all terms (including the zero on the right) by the current lead coefficient. The lead coefficient is the number in front of the x^2 .

$$\frac{3x^2}{3} + \frac{12x}{3} - \frac{6}{3} = \frac{0}{3}$$

$$1x^2 + 4x - 2 = 0$$

Move the constant term to the right side of the equation.

$$1x^2 + 4x - 2 + 2 = 0 + 2$$

$$1x^2 + 4x = 2$$

Step 4:

Complete the square

a). Take half of the coefficient next to the x-term. (Note: the number you obtain is used later when you factor).

$$\frac{1}{2} \text{ of } (4) \text{ is } (2)$$

b). Square the number you obtained in the last step.

$$(2)^2 = 4$$

c). Add the result to BOTH SIDES of the equation.

$$1x^2 + 4x + 4 = 2 + 4$$

Step 5:

Factor the polynomial on the left side of the equation and combine the numbers on the right side.
Note: the number needed to factor the polynomial is always obtained in Step 5a.

$$1x^2 + 4x + 4 = 2 + 4$$

$$1x^2 + 4x + 4 = 6$$

$$(x+2)(x+2) = 6$$

$$(x+2)^2 = 6$$

Step 6: Solve the equation.

Method: lead coefficient of 1 and do NOT move the constant term.

Solve: $3x^2 - 3x - 2x + 2 = -17x + 8$

Step 1:

Use algebra to simplify both sides of the equation.

$$3x^2 - 3x - 2x + 2 = -17x + 8$$

$$3x^2 - 5x + 2 = -17x + 8$$

Step 2:

Again, use algebra to write the equation in standard form: a polynomial on the left side of the equation and a zero on the right. The polynomial should be listed in descending powers: the highest power first and the constant last.

$$3x^2 - 5x + 2 = -17x + 8$$

$$3x^2 - 5x + 2 - 8 = -17x + 8 - 8$$

$$3x^2 - 5x - 6 = -17x$$

$$3x^2 - 5x - 6 + 17x = -17x + 17x$$

$$3x^2 - 5x + 17x - 6 = 0$$

$$3x^2 + 12x - 6 = 0$$

Step 3:

If the lead coefficient is NOT the number 1, then divide all terms (including the zero on the right) by the current lead coefficient. The lead coefficient is the number in front of the x^2 .

$$\frac{3x^2}{3} + \frac{12x}{3} - \frac{6}{3} = \frac{0}{3}$$

$$1x^2 + 4x - 2 = 0$$

Step 4: Complete the square

a). Take half of the coefficient next to the x-term. (Note: the number you obtain is used later when you factor).

$$\frac{1}{2} \text{ of } (4) \text{ is } (2)$$

b). Square the number you obtained in the last step.

$$(2)^2 = 4$$

c). Add AND subtract the result to the right of the term with the letter ("x") in it.

$$1x^2 + 4x + 4 - 4 - 2 = 0$$

d). Using parentheses, group the first three terms on the left side; do the same with the two constant terms.

$$(1x^2 + 4x + 4) + (-4 - 2) = 0$$

Step 5: Factor the polynomial (the first three terms) and simplify within the second set of parentheses.

$$(x+2)(x+2) + (-4-2) = 0$$

$$(x+2)^2 + (-6) = 0$$

Step 6: Now move the constant term and solve for x:

$$(x+2)^2 + (-6) + 6 = 0 + 6$$

$$(x+2)^2 = 6$$

Step 7: Solve the equation

Method: Do not divide by the lead coefficient. Complete the Square inside parentheses.

$$\text{Solve: } 3x^2 - 3x - 2x + 2 = -17x + 8$$

Step 1:

Use algebra to simplify both sides of the equation.

$$3x^2 - 3x - 2x + 2 = -17x + 8$$

$$3x^2 - 5x + 2 = -17x + 8$$

Step 2:

Again, use algebra to write the equation in standard form: a polynomial on the left side of the equation and a zero on the right. The polynomial should be listed in descending powers: the highest power first and the constant last.

$$3x^2 - 5x + 2 = -17x + 8$$

$$3x^2 - 5x + 2 - 8 = -17x + 8 - 8$$

$$3x^2 - 5x - 6 = -17x$$

$$3x^2 - 5x - 6 + 17x = -17x + 17x$$

$$3x^2 - 5x + 17x - 6 = 0$$

$$3x^2 + 12x - 6 = 0$$

Step 3: Using the lead coefficient (the number in front of x^2) as a common factor, factor it from the polynomial and place it in front of a set of parentheses.

$$3(x^2 + 4x - 2) = 0$$

Step 4: Complete the square

a). Take half of the coefficient next to the x-term. (Note: the number you obtain is used later when you factor).

$$\frac{1}{2} \text{ of } (4) \text{ is } (2)$$

b). Square the number you obtained in the last step.

$$(2)^2 = 4$$

c). Add AND subtract the result to the right of the term with the letter ("x") in it. But do it inside of the parentheses.

$$3(x^2 + 4x + 4 - 4 - 2) = 0$$

d). Simplify, but leave the polynomial unfactored for now.

$$3(x^2 + 4x + 4 - 4 - 2) = 0$$

$$3(x^2 + 4x + 4 - 6) = 0$$

Step 5: Distribute inside the parentheses. Multiply the polynomial as a quantity with parentheses. Do the same for the constant, but take into account the sign obtained from multiplying, e.g. positive times negative is negative.

$$3(x^2 + 4x + 4 - 6) = 0$$

$$3(x^2 + 4x + 4) - 3(6) = 0$$

$$3(x^2 + 4x + 4) - 18 = 0$$

Step 6: Move the constant and divide by the number in front of the parentheses.

$$3(x^2 + 4x + 4) - 18 = 0$$

$$3(x^2 + 4x + 4) - 18 + 18 = 0 + 18$$

$$3(x^2 + 4x + 4) = 18$$

$$\frac{3(x^2 + 4x + 4)}{3} = \frac{18}{3}$$

$$(x^2 + 4x + 4) = 6$$

Step 7: Factor the polynomial. Use the factor you obtained when you divided the coefficient of "x" by two. [See Step 4a above.](#)

$$(x^2 + 4x + 4) = 6$$

$$(x + 2)(x + 2) = 6$$

$$(x + 2)^2 = 6$$

To solve the equation means to find x. Now solve for x

a). Square root both sides of the equation. Use a plus/minus sign on the right side in front of the radical.

$$(x + 2)^2 = 6$$

$$\sqrt{(x + 2)^2} = \pm\sqrt{6}$$

$$(x + 2) = \pm\sqrt{6}$$

b). Get "x" alone. Move any constant term to the right side, change its sign, and place it in front of the plus/minus sign. If possible, simplify the radical by removing perfect root factors, e.g. the square root of 25 is 5.

$$(x + 2) = \pm\sqrt{6}$$

$$x + 2 = \pm\sqrt{6}$$

$$x + 2 - 2 = -2 \pm \sqrt{6}$$

$$x = -2 \pm \sqrt{6}$$

c). Notice that there are 2 answers.

$$x = -2 + \sqrt{6} \text{ or } -2 - \sqrt{6}$$