GRAPH OF A RATIONAL FUNCTION

Find vertical asymptotes and draw them. Look for common factors first. Vertical asymptotes occur where the denominator becomes zero as long as there are no common factors.

Find the horizontal asymptote, if any, and draw it. A horizontal asymptote may be found using the exponents and coefficients of the lead terms in the numerator and denominator. Please see "Horizontal Asymptotes and Lead Coefficients" below.

Pick two or three x-values to the left and right of each **vertical asymptote**. If there are no vertical asymptotes, then just pick 2 positive, 2 negative, and zero. Put these values into the function f(x) and plot the points. This will give you an idea of the shape of the curve. You will quickly see whether the graph goes above or below the ***horizontal asymptote***.

Find the y-intercept by letting $x = 0$. Put "0" into the function, do the arithmetic and get the y-value. Plot the point (0, y). This will show you where the curve crosses the y-axis.

Find any x-intercepts by letting $y = 0$. Replace f(x) by "0", then solve for x. Plot these points as (x, 0). This will show you where the graph crosses the x-axis. By the way, when you set $y = 0$ it is possible that you will get imaginary answers for x. Since we can not graph $a + bi$ on the "real plane", this means there are no x-intercepts.

Finally, connect all of the points (x, y) with a smooth curve rather than choppy, straight line segments.

Horizontal Asymptotes and Lead Coefficients

RATIONAL FUNCTION: $f(x) = \dfrac{cx^a + c_2x^{a-1} + c_3x^{a-2} + \ldots + c_n}{dx^b + d_2x^{b-1} + d_3x^{b-2} + \ldots + d_n}$

Focus on the lead (first) term in the numerator and the lead term in the denominator.

$$f(x) = \frac{cx^a + \ldots}{dx^b + \ldots.}$$

Look at the exponent in the top (this is "a") and the exponent in the bottom (this is "b").

1, The equation of the HORIZONTAL ASYMPTOTE will be $y = \dfrac{c}{d}$
if the top exponent is the same as the bottom exponent, i.e. when $a = b$ use the lead coefficients.

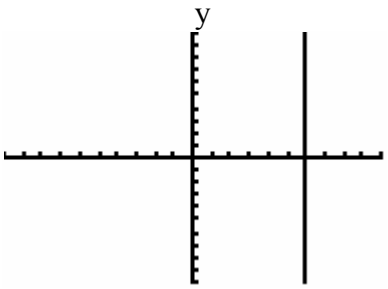
2. The equation of the HORIZONTAL ASYMPTOTE will be $y = 0$
if the top exponent is smaller than the bottom exponent, i.e. when $a < b$ the degree of the denominator will be larger than the numerator, so the bottom becomes much larger than the top as $x \to \infty$,

3, There will be NO HORIZONTAL ASYMPTOTE when the top exponent is larger than the bottom one. In other words, the degree of the numerator will be larger than the denominator, so there will always be some degree of "x" in the top. Thus when x gets very large, y will also get very large. In a sense, as positive "x " gets very large, the curve approaches $+\infty$ or $-\infty$ depending on the signs of the lead coefficients. If the signs are the same, the curve goes to $+\infty$ when x also goes to $+\infty$. Remember negative divided by negative is positive, and positive divided by positive is also positive. If the signs of the lead coefficients are opposite and <u>*positive*</u> x gets very large, the curve goes to $-\infty$ because negative divided by positive is negative and vice-versa.

Example 1:

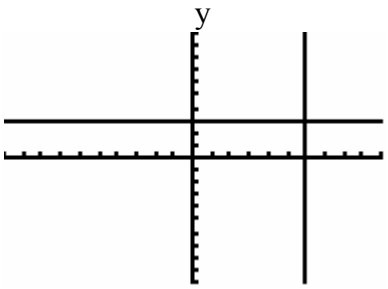
Graph by hand: $f(x) = \dfrac{3x}{(x-6)}$

Find vertical asymptotes and draw them. Vertical asymptotes occur where the denominator becomes zero. There is a vertical asymptote at $x = 6$.



Find the horizontal asymptote, if any, and draw it. A horizontal asymptote may be found using the exponents and coefficients of the lead terms in the numerator and denominator.

In $f(x) = \dfrac{3x}{(x-6)}$ the exponents in the top and bottom are both 1. The coefficient of the top is 3 and the coefficient of the bottom is 1, so there is a horizontal asymptote at $y = \dfrac{3}{1} = 3$



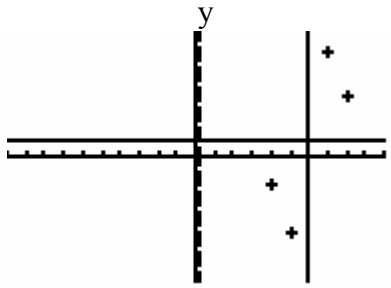
Pick two or three x-values to the left and right of each **vertical asymptote**.
You will quickly see whether the graph goes above or below the ***horizontal asymptote***.

There is a vertical asymptote at $x = 6$, so to the left pick x = 4 and x =5 because they are close to the asymptote.

When x = 4, $f(x) = \dfrac{3(4)}{(4-6)}$ will be -6. When x = 5, $f(x) = \dfrac{3(5)}{(5-6)}$ will be -15.

Try x = 7 and x = 8 to the right. The points produced will be $(7,21)$ *and* $(8,12)$

y



Find the y-intercept by letting x = 0 .

When x = 0, $f(x) = \dfrac{3(0)}{(0-6)} = 0.$

So the curves passes through the origin (0,0).

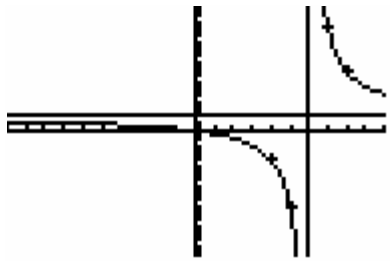
Find any x-intercepts by letting y = 0

When $f(x) = \dfrac{3x}{(x-6)} = 0$ we also get x = 0.

So the x-intercept and y-intercept are the same.

Finally, connect all of the points (x, y) with a smooth curve rather than choppy, straight line segments

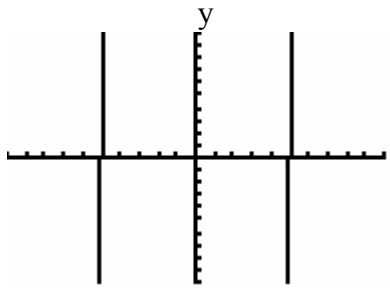
y



Example 2:

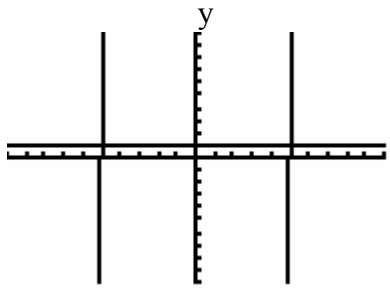
Graph by hand: $f(x) = \dfrac{x^2}{(x^2 - 25)}$

Find vertical asymptotes and draw them. Vertical asymptotes occur where the denominator becomes zero. There are vertical asymptote at $x = 5$ *and* -5.



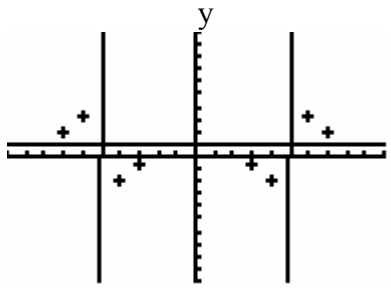
Find the horizontal asymptote, if any, and draw it. A horizontal asymptote may be found using the exponents and coefficients of the lead terms in the numerator and denominator.

In $f(x) = \dfrac{x^2}{(x^2 - 25)}$ the exponents in the top and bottom are both 2. The coefficient of the top is 1 and the coefficient of the bottom is 1, so there is a horizontal asymptote at $y = \dfrac{1}{1} = 1$



Pick two or three x-values to the left and right of each **vertical asymptote**.
You will quickly see whether the graph goes above or below the ***horizontal asymptote***.

There is a vertical asymptote at $x = -5$.
To the left of it pick x = -7 and x = -6 because they are close to the asymptote.
To the right, try x = -4 and x = -3.
To the left of the other asymptote, "$x = 5$", pick x = 3 and x = 4.
To the right of "x = 5" pick x = 6 and x = 7.
This will produce the points:
$(-7, 2.04)$, $(-6, 3.27)$, $(-4, -1.78)$, $(-3, -.56)$, $(3, -.56)$, $(4, -1.78)$, $(6, 3.27)$, $(7, 2.04)$

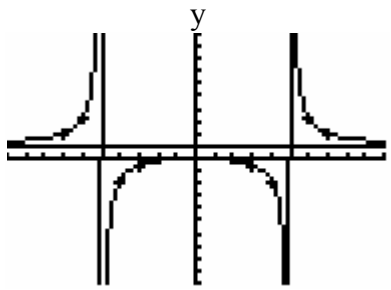


Find the y-intercept by letting x = 0 .

When x = 0, $f(x) = \dfrac{(0)^2}{(0^2 - 25)} = 0.$

So the curves passes through the origin (0,0).

Find any x-intercepts by letting y = 0

When $f(x) = \dfrac{x^2}{(x^2 - 25)} = 0$ we also get x = 0.
So the x-intercept and y-intercept are the same.

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