



## FACTORIZING A TRINOMIAL

To factor means to rewrite a single expression as a multiplication problem.

For example,

when we're asked to factor the number 12, people respond with answers such as (2) times (6), (3) times (4), or (1) times (12).

"12" is a single expression; (2) times (6) is a multiplication problem written as (2)(6). So one way to "factor" 12 is to write it as (2) times (6). This is factoring.

When we "factor"  $x^2 + 5x + 6$  as  $(x + 2)$  times  $(x + 3)$ , we are rewriting the single expression (the polynomial:  $x^2 + 5x + 6$ ) as a multiplication problem,  $(x + 2)(x + 3)$ .

### Methods of Factoring Polynomials: Trinomials.

**Factor:**  $x^2 - 7x + 12$

Goals: We must answer 3 questions to factor any trinomial (a polynomial with 3 terms):

1. "What times what" will create the first term of the polynomial?

These become the candidates for the first terms of each factor:

( **First 1** .....)(**First 2**.....)

2. "What times what" will create the last term?

These become the last term candidates:

(.....**Last 1**)(.....**Last 2**)

3. Next multiply the two nearest terms (the inner terms). Then multiply the two most distant terms (the outer terms). Add the two answers. If the factors are correct, the result should be the second term in the original polynomial.

**In our example,  $x^2 - 7x + 12$**

1. "What times what" will create  $x^2$  ?

Answer: (x) times (x) .

So we write ( **x** .....)( **x** .....)

## 2. "What times what" will yield 12 ?

Answer: (1)(12), (2)(6), or (3)(4).

Looking closely at the choices, which pair is most likely to produce the middle term of  $7x$  in the original polynomial? Probably (3)(4), so we'd have:  $(x \dots 3)(x \dots 4)$ . We'll check it in the next step.

### 3. Add the inner and outer products.

The **3** in the first parentheses and the **x** in the second are the two *nearest* terms. When multiplied, they produce  $3x$  called the "*inner product*".

The **x** in the first parentheses and the **4** in the second parentheses are the *farthest* terms apart. Their product is  $4x$  called the "*outer product*".

Adding  $3x$  and  $4x$  produces  $7x$ , but we want the result to be *negative*.

This will occur when both the 3 and the 4 are negative.

So we have:

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

We have factored the polynomial since it is now written as a multiplication problem.

## ANOTHER EXAMPLE:

**Factor:**  $24x^2 - 14xy - 3y^2$

### 1. "What times what" creates $24x^2$ ?

Answers:  $(1x)(24x)$ ,  $(2x)(12x)$ ,  $(3x)(8x)$ ,  $(4x)(6x)$ .

So when we write the first terms in the factors they might look like:

$(1x\dots\dots)(24x\dots\dots)$  or  $(2x\dots\dots)(12x\dots\dots)$  or .....

### 2. "What times what" creates $3y^2$ ?

Answers:  $(1y)(3y)$

The last terms in the factors will be:  $(\dots\dots 1y)(\dots\dots 3y)$

### 3. What is the sum of the inner and outer products?

Here we have to be creative, use trial and error, and some arithmetic. If we pick the factors:  $(1x + 1y)(24x - 3y)$ , the inner product is  $(1y)(24x) = 24xy$  and the outer product is  $(1x)(3y) = 3xy$ .

The sum of  $24xy + 3xy$  is  $27xy$ . We want  $-14xy$  (see the second term of the original polynomial).

If one term was negative and one was positive, we'd still get  $-24xy + 3xy = -21xy$  (this is the trial and error part).

After some more playing with different combinations, we find:

$$(4x - 3y)(6x + 1y)$$

Here, the inner product is  $(3y)(6x) = 18xy$  and the outer product is  $(4x)(1y) = 4xy$ .

While the sum of  $18xy$  and  $4xy$  is  $22xy$ , if the 18 were negative the sum would be  $-18xy + 4xy = -14xy$  (*which is what we want!*)

Looking at the *inner product*, the way we get  $-18xy$  is to have a negative  $3y$  and a positive  $6x$ :  $(-3y)(6x)$  is  $(-18xy)$ .

$$(4x - 3y)(6x + 1y)$$

Looking at the *outer product*, to get  $4xy$ , the  $4x$  and  $1y$  both have to be positive.

$$(4x - 3y)(6x + 1y)$$

Thus we have,

$$24x^2 - 14xy - 3y^2 = (4x - 3y)(6x + 1y)$$

The polynomial is now factored since it is now written as a multiplication problem.